## Comparing Gains to Income, <br> Converting Yields to Returns

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## Introduction

## The Challenge

There is no more basic investment decision determining future returns than top-down allocation, putting individual investments into an overall balance and structure. The decisions are inescapable. Even the bottom-up investor picking one chosen stock at a time ends up with an allocation - a balance between investments and investment types. To not decide is to decide. Allocations are more critical to future performance than individual stock selections.

The most fundamental allocation decision is between owning debt and owning equity, between fixed income and a goal of price appreciation. How do we compare these two alternatives in their most basic and pure form, uncontaminated by the other? What data should we be looking at and what should we be thinking when making these most basic of investment decisions?

## Making Investment Choices

The two main considerations in making an investment decision are the certainty of producing a positive return over expected time frames, and deciding whether to look for returns from income and/or appreciation.

We will contrast the certainty or volatility of investments made primarily or exclusively for price appreciation, and investments made primarily for income.

The second and primary interest is to find a way to formulate income yields with a number directly comparable to returns. If a stock has an annual return of $10 \%$ over ten years, how does that compare to a preferred stock having a $10 \%$ annual yield? The answer is important in making allocation decisions such as between equities with returns based on price or valuation, debt instruments with fixed income, or in choosing investments for gain and income. Most investors don't stop to think about how a number for yield may not be the same as a return number based on price changes. Is there a formula or formulation that equates yield to return?

The choice between investments for their income and for their possible gain is properly based on volatility probabilities and relative returns. Here we will examine volatility for each type of investment and then drill down and look at the details of how each is calculated, and why it may not be accurate to directly compare a $10 \%$ yield to a $10 \%$ return. Typically, a reported annual $10 \%$ dividend yield is based on the original cost, while an annual $10 \%$ return is based on the changed valuation at the beginning of each year.

Most investments may have a combination of gains from price variation and income such as dividends, interest, or operating profits. Investments designed primarily for price gain often have dividends in the background. Investments intended primarily as fixed income also have a spread between the purchase price and at some point, a sale. Some income investments have variable income. Different investment vehicles combine these contrasting factors in myriad ways. All of that complicates the analysis and picture painted here, intended to understand the two basic types of investment, gains or losses and income. We want here to look at the pure types, one relying on changes in price or valuation and the other on ongoing fixed income. They are calculated differently. Once we have that understood with appropriate metrics, we can consolidate our methodology and considerations for more complex investments. Here we will mostly ignore the buying and selling of fixed income for gain or loss and mostly ignore the variety of variable and conditional income instruments.

If you are relatively new to the details of calculating investments returns, or find some of the following material confusing, the Appendix gives some context and background in an outline form.

## Valuation-Based Investments: Volatility and Probabilities

For a stock with an annual 10\% return over ten years, the picture of a simple compound annual growth rate (CAGR) is shown in Table 1. In real life valuation changes are not that consistent, but initially we want a simpler model to contrast the distinctions between income and gains.

## Table 1

Valuation-based Investment


In our Table 1 example, each year the $10 \%$ change is based on the preceding cumulative changes to give the geometric pattern. Until the sale, gains are unrealized (hypothetical) rather than real or cash in the account. If one is investing for the entire ten-year period, variations in price really don't matter unless one is using the investment as collateral for a loan, or the purpose is for emotional satisfactions such as pride or anxiety in watching the price.

Our thinking is dependent upon what we see and the data we take in. We change our thinking by what we watch. Why do investors track price changes if they are going to buy and hold?

## Return Distributions

Market prices are not that consistent, and volatility is hard to ignore.
Market returns for the past ninety-six years are shown in Table 2. The cumulative effect is up but there is wide variability year to year and over consecutive years. Annual returns are positive $73 \%$ of the time (see Table 3), but that is not predictable except in a general way, and which years are up and how much is quite unpredictable.

Table 2


Table 3
Table 3 shows the same data sorted by the amount of return rather than by year.


The median of $13.9 \%$ informs us that by chance, about half the time our annual returns are expected above that amount and about half the time below. However, a median is not the same as a compounded annual growth rate (CAGR). Besides the frequency, the other question is how the annual return percentages below the $13.9 \%$ compare to those above. An infrequent but severe decline will have an outsized impact. In the chart, the dotted trendline is fairly straight except for the lowest and highest eight years. Because of the way math works with a decline of $50 \%$ needing an increase of $100 \%$ to come back equal, we can't view the negative returns mathematically in the same way as the positive returns.

Based on the returns from Table 2, one dollar invested in 1928 became $\$ 6,976$ at the end of 2022, equal to a compound annual growth rate (CAGR) of 9.66\%.
Table 4


Table 5 shows the last thirty years having a CAGR rate of 8.56\%.


Most years had returns considerably at variance from the average. (Standard deviation .194)
Because of the volatility, is there any merit or meaning in looking at last year's annual return, or even returns over the past several years? Tables 4 and 5 make it look like a fairly reliable momentum, in contrast to Tables 2 and 3. Can we predict a return based on a trend extending from the previous year? If we look at correlations between a return from one year to the next year over the 96 years, the correlation is $-1.5 \%$, or about as random and meaningless as one could find. One should not look at returns from the last year in expecting better or worse returns for the next year.

How about average returns over the past five years? The five-year separate sequences have a negative correlation of $-19 \%$. The negative correlation meaning reversals is interesting, but the number is not large enough to be significant. Five-year returns are also statistically meaningless in predicting the amount of an annual return.

Is the market example given here irrelevant if rather than buying general market funds, one can presumably select and buy individual stocks with exceptional performance?

I have done extensive analysis of returns over varying time periods using a decision-tree software form of artificial intelligence. Up to a million rows of data and eighty columns or variables containing both fundamental and price pattern (technical) data are assembled. The software uncovers patterns using individual variables or combinations of variables with different ranges for each variable. Inevitably, returns of the overall market were found to be the variable most statistically correlated with price returns for any stock. If the market went up, most stocks predictably went up. Being contemporaneous rather than prior in time, that finding is not very useful except to point to the value of using funds or to use a collection of stocks and then expect market returns. Variable combinations other than market fluctuations were also predictive, but less so and rarely significantly so for a large enough sample and over multiple time periods of varying durations.

Can returns be predicted by factors other than prior returns? If so, we should look to those factors rather than history. However, with the sophisticated and highspeed algorithms predominant in the market, almost all those predictable factors are incorporated in the market. The market does reflect short-term traders, long-term investors, and every duration in between. To the extent one's holding period is outside the market balance, there might be opportunity.

Investing for price appreciation is a lot like spinning a roulette wheel. One should think in terms of probability as much as the return or number from the last spin. (One might also ask if the house has rigged the wheel.)

## Simulations

Simulations are a way to look at probability in conjunction with returns.

## Table 6



To explain Tables 6 and 7, assume we pick the annual return shown in Table 2 from a randomly selected year between 1928 through 2022 and assign that to next year applying it to $\$ 100,000$, then did it again and assigned it to the second year out times the value from the end of the preceding year, and continued the pattern for ten years' of returns. We do that a hundred times and break the average results into quintiles. Based on historical random returns, looking at the solid lines in the Table 6 we see a $20 \%$ probability of the $\$ 100,000$ becoming about $\$ 400,000$ and another $20 \%$ probability of being close behind. The lowest $20 \%$ probability comes in at about $\$ 150,000$. The line with the shorter dashes is based on a CAGR of $9.0 \%$ which is less than $60 \%$ of the probabilities, matches $20 \%$ and is more than the lowest $20 \%$. The line with the longer dashes is a $5 \%$ probability of the lowest returns. Taking a CAGR return from past years is not a very accurate metric for projecting future returns.

Table 7


Running the hundred simulations again, we happen to see less variation in Table 7 between the probability quintiles, and all of them better than the CAGR of $9 \%$. A simple keystroke reruns the simulations, revealing the variation in different randomly selected annual returns. (The Excel file is found under Planning at WenzelAnalytics.com.)

We tend to look at return numbers with much more precision than is rational. We should pay as much attention to probabilities as to return.

Before comparing price appreciation returns to income returns, we need to examine how we measure income returns and to what we are comparing. The next section on calculating income returns will include some comparisons to price appreciation in how returns are calculated. If the implications of the alternative ways to calculate yield seem overly detailed and technical, feel free skip ahead to the section on the overall comparisons of price appreciation to income.

## Calculating Income Returns

## How are fixed income and price appreciation very different?

We easily see the label of fixed income without realizing what it means. Since the income is fixed, we know what it will be for the next year or longer. Knowing what we paid for the position, and knowing the future income stream from dividends or interest, we can calculate with relative certainty a Yield on (original) Cost, ignoring dividends invested in the same or other positions. It doesn't change until the position is called or sold. I say relative certainty assuming we have done our due diligence in selecting a quality investment and a deferred dividend or bankruptcy are unlikely. Based on that understanding we can make concrete plans and projections without the anxiety that comes with uncertainty. The dividends are realized or real, meaning cash in the account.

If we are buying for income or wanting to measure returns strictly on the income component, any price metric is a contamination. After purchase, price introduces the variability of what others would pay for the investment and is irrelevant to the income. Any pricing of income securities after the initial purchase introduces characteristics of the opposing asset class, investments purchased with a goal of appreciation.

Fixed income is realized, meaning money collected and in the account. It is very different than a market valuation which is unrealized. With portfolios intended for price appreciation we are dependent upon an uncertain future. Reports with specific performance numbers focus on the past rather than the future as can be done with fixed income. Today's valuation, or the valuation on any periodic report, is hypothetical meaning that if everything was sold, that is approximately what would be realized. But if we didn't sell everything today, or on the date of the report, time moves on and the numbers and returns based on unrealized gains or losses are history and largely irrelevant. The current valuation functions mostly to give us pride or distress, or to make assumptions about the future which may be valid or invalid. Will a trend continue, or will it reverse? No other decision is more critical to the success of any price-dependent strategy than decisions about reversals.

## Getting a Yield Number Equivalent to Return

When evaluating investment results it would be nice to have a results number from income that can be directly compared to a results number from price appreciation. Unfortunately, the language is confusing as there are many ways to calculate yield and return, and the terms are often read and interpreted without an understanding of the assumptions that went into the calculations. To have an investment producing periodic cash flow is different than an investment with an initial cost and no other tangible or cash flow result until it is sold. Looking at the combination of income and gains from an investment only confuses the matter further.

This inquiry began as I questioned the accuracy and utility of reporting fixed income returns as yield based on original cost, referred to as Yield on Cost. Yield on Cost is attractive because it doesn't have the volatility of returns connected to price variations. What we paid for a position doesn't change, nor does the dividend prescribed by the prospectus, thus giving a consistent return. My discomfort, prompting the analysis and redefinitions of yield which follows, stems from the fact that Yield on Cost ignores the time value of consequent dividends and dividend reinvestment. Yield on Cost gives an arithmetic chart or progression. The actual income yields are geometric. (Ignoring for now any consideration of realized or unrealized gains or losses upon sale.) Arithmetic or geometric (CAGR) calculations have the same result for the first year. There is a considerable difference a few years out. Indeed, ten years out the difference for a $10 \%$ yield is $30 \%$ as shown in Table 8.

Table 8
Income-based Investment


The orange bars are the arithmetic returns from dividends of \$100 each year for ten years, coming to $\$ 1,000$ dividends added to the investment of $\$ 1,000$. The grey tops of the bars show the added value of the geometric progression coming from dividend returns on dividends. As we will examine below, this is the same whether the dividends are reinvested in the same investment or other more attractive positions when the dividends are received.

A Compound Annual Growth Rate (CAGR) smooths out the in-between returns into one number, which when applied to each successive year or period, connects the beginning and end dates with one return number rather than a possible series of divergent returns. What is particularly relevant for CAGR returns is that it gives a metric that can be accurately applied to just income, to gains and to the combination of income and gains, known as Total Return. It gives us a metric for comparing yields to price appreciation returns.

However, the orange bars in Table 8 show how Yield on Cost can be an inadequate metric for fixed income returns. Yield on Cost fails to include geometric reinvestment returns. It fails in other ways as well. What criteria should apply when formulating a way to measure fixed income returns?

## Criteria for a Yield Calculation

Ideally a yield calculation should:

1. Give a number that one could directly compare to price-appreciation gains.
2. Reflect the geometric compounding of dividends.
3. Have an accurate yield number for each position, easily aggregated for an entire portfolio.
4. Be adjusted and accurate for each progressive year a position is held.
5. Should account for the time value of dividends through some method of calculating present value for discounted future cash flows.
6. Be independent of ongoing price variations. An eventual gain or loss on a call, maturity or sale should be a separate calculation.

Dividend reinvestments are either calculated at the original cost, which is not accurate since prices change and reinvestments in the same or other positions will have different prices and yields, or the reinvestments will be dependent upon future and unknown prices. Doing historical reporting allows for known prices on reinvestments but presents other complications as will be detailed below.

How close to those ideals can we get with a creative formulation, or a proper understanding of formulations currently in use? Before getting into formulating specific ways of calculating yields and returns, and discussing their respective merits, some context is relevant to understanding how and why such calculations are useful.

## Yield and Return Calculations

The yield on an income investment is calculated as the annual dividend or interest amount divided by the valuation. A security may be purchased at par or at a market price above or below the initial par offering. On fixed income, yield stays the same, since neither the cost nor the income change. When the first income is received, the yield number is the same as a gain of the same amount from an investment simultaneously bought and sold. However, as the dividends proceed, how are the dividend time values calculated so as to make a proper comparison to an eventual investment gain based on price variation? How do we get the gray portions of the bars on Table 8 to get a CAGR return comparable to price appreciation returns?

For gains and losses, realized return is the percent change from purchase to sale, and then annualized only if held for more than a year. Industry reporting requirements specify that if the holding is for less than a year, the result is identified as a percent change rather than an annualized return. For an unrealized return where the position has not been sold, the return is calculated the same way, either since purchase or for a given time period. The return calculation only relates to the delta between two very specific points in time, the beginning of the period and the end. Even with modest volatility, changing the dates by even a few days can cause significant differences in the calculated return.

For dividends calculations, the numerator is the amount of dividends, just like price change is for gains and losses. While the denominator for gains and losses is the beginning valuation, there are several denominator alternatives when computing income results.

Yield calculations depend upon whether dividends are reinvested in the same position, invested in another position, or withdrawn.

To illustrate, let us look at a preferred stock having a par value of $\$ 25$ and 50 shares purchased for $\$ 20$ each, creating the same $\$ 1,000$ investment as in Table 8 . The stock (actually debt rather than equity) has a contractual dividend of $8 \%(x \$ 25)$ or $10 \%$ yield if purchased at the discounted price of $\$ 20$. How does a $10 \%$ yield match up against a $10 \%$ annual return from an investment intended for gain from a sale? How do we compare a steady cash flow to an end capital gain with no intervening cash flow?

## Accounting for Successive Dividends

To get an income-based number comparable to price-appreciation returns, one needs to account for the successive dividends. The value of dividends can be handled in different ways.

1. Dividends can be subtracted from the original cost as reducing Out of Pocket investment. This reduces the cost or basis for the next period calculation.
2. A Yield on Cost ignores the time value of dividends.
3. Dividends can be reinvested in the original investment calculation as additional shares.
4. Dividends can be reinvested in new and different positions.

## 1. Dividends Reducing Out of Pocket

My original thinking on this project was to calculate a yield or return based on Out of Pocket. Money is put into an investment. Money is returned back in the form of dividends. Each year the yield goes up as it is larger relative to the Out of Pocket money remaining in the investment or the denominator in the calculation. A refinement would be to adjust the value of the dividend according to the time value from the original investment or the discounted future value. This concept is shown in Table 9.

## Table 9

Yield from Out-of-Pocket


The Out of Pocket (OOP) term and calculation is used by the software I use for charting and return calculations (Fund Manager). Out of Pocket refers to what has been withdrawn compared to original cost. Out of Pocket is similar to an adjusted basis, which is more of a tax term. For reasons obvious in Table 9, the software calculates dollar amounts for Out-of-Pocket, but not yields. The yield calculation obviously doesn't work when there is a negative amount of money left in the investment and it gives distorted yields prior to that. The fallacy is that while the Out-of-Pocket declines, the original investment in the example is still there producing the $10 \%$ dividend.

The framework is relevant if someone is withdrawing the dividends rather than reinvesting them, and who wants to think in terms of dollars rather than yields.

## 2. Ignoring Yields on Dividends

Successive dividends can accumulate, ignoring the time value of when they were received. The numerator and denominator remain the same for each successive period. Yield on Cost is shown in Table 10. Unless there are withdrawals, a $\$ 1,000$ investment becomes $\$ 2,000$ in ten years.

Table 10


Yield on Cost is a common way to report yield on preferred stocks and has been how Wenzel Analytics has reported dividend income. Dividends are divided by original cost. As already noted, this gives arithmetic returns for the position since reinvested dividends are not credited to the specific original investment.

Yield on Cost ignoring reinvestments is applicable to investors withdrawing the dividends. This dovetails well for investors needing the income for lifestyle or other investment needs, such as paying a mortgage that has a lower interest rate than the preferred stocks.

For returns on a portfolio rather than for a specific position, instead of aggregating normalized returns on individual positions, the total dividends paid are divided by the total of original costs.

## Discounted Future Value

One shortcoming of Yield on Cost is in ignoring the discounted value of the dividend when having to wait successive years to receive it. To give an extreme example from price appreciation of the time value between acquisition and the period of calculation, imagine having purchased Apple in 2002 for the split-adjusted price of $\$ .30$. At the beginning of 2022 it was trading for $\$ 172$, having gone up $\$ 40$ in the previous year for a return of $30 \%$. If one did a "Return on Cost" it would be $13,333 \%$ for the year.

Yield on Cost has dividends based on cost, not on the reinvested value of dividends adding to the value of the investment. In our extreme Apple example, it is like the $30 \%$ return applies to the $\$ .30$ instead of the $\$ 132$.

The Yield on Cost is accurate the first year assuming annual calculations rather than quarterly when the dividends are usually paid. For our illustration purposes, there are no quarterly reinvested dividends, and the acquisition is at the beginning of the reporting period. The inaccuracy of the yield increases each year as the time increases between acquisition and the current reporting period. The value of a dividend for which we wait ten years is less than the value of a dividend received now. To be accurate one would apply a discounted future value to the dividend stream. The value of a future dividend subtracts from the long-term yield on the original investment while the value of reinvested dividends adds to the yield. The reinvested dividends increase the base amount to which the yield applies, while the discount on future dividends applies only to the dividends to be received or to the time value of dividends as paid.

These dynamics are illustrated in the Table 11 example. Reinvested dividend calculations, elaborated in the next sections, are included in Table 11 to show how they combine with the discounted cash flow and Yield on Cost calculations. Column B, shown on the chart with the dark blue bars, is the dollar amount by year of reinvesting dividends using the parameters being mostly used throughout this paper. Column C is Yield on Cost as shown in Table 11 and on the second orange bar for each year. Column D in gray reflects the future discounted values of having to wait for the dividends. You can see how the reinvested dividend increases outpace the future value decreases. The net between reinvested dividends adding value and the discounted future value subtracting value is represented in Column E and with the fourth light orange bar for each year.

Table 11


Column F in Table 11, shown by the blue line and the scale on the right, is the CAGR from inception to each year. The line shows the difference between the arithmetic returns of Yield on Cost and geometric returns of reinvested dividends, adjusted for the discounted future values. While the yield decrease from $10 \%$ to $8.1 \%$ in our example is not severe, it is significant. Obviously, from looking at the chart short-term investments are more attractive. (This is in contrast to investments for capital gains, where because of the probabilities involved, it is better to diversify over a long time frame and a number of positions.)

A disadvantage of preferred stocks in this regard is that they often are not called when callable. Fortunately, the blue line in our example bottoms at about year seven and slopes up thereafter.

Table 12 is the same as Table 11 except for a yield of $7 \%$ instead of $10 \%$. Note the differences. The CAGR line has a different scale, shown by the number on the right, in order to have the line start with the bars.

Table 12


As it happens, reinvesting quarterly dividends in new positions minimizes the average duration of holdings. I worked out a table by year for each of ten years listing how many positions were one year old, two years, three years, etc. Because of the continual reinvesting of dividends in new positions, sixty percent of the hypothetical portfolio of 230 positions after ten years were held three years or less. Forty-eight percent were held two years or less.

Obviously, including the discounted future dividend values in the calculation gives a lower yield. Less obvious is that if that is done for dividends, it should be done for gains as well. For gains a future value for the one-time sale would be the comparable calculation rather than for a series of payments. To do a future value as a projection based on goals is a guessing game of hope. To do it retrospectively is an arbitrary exercise in picking a rate for the discounted future value. If the rate matches the CAGR, the gains disappear. We calculate the discounted future value on income because we can. The possibility can become a comparative disadvantage if not equally applied.

## 3. Reinvesting Dividends in the same Position

We will next discuss the difference in yields/returns when incorporating reinvested dividends, or a Compound Annual Growth Rate (CAGR). To simplify the analysis, we will ignore the discounted future value considerations described above. The charts below give another perspective on the contrast in returns between arithmetic, such as Yield on Cost, and compound returns from reinvested dividends.

Table 13

| Arithmetic or Yield on Cost$1,000$ |  |  |  |  |  |  | 3,000 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Yr | 2\% | 4\% | 6\% | 8\% | 10\% | 12\% |  |  |  |  |  |  |  |  |
| 1 | 1,020 | 1,040 | 1,060 | 1,080 | 1,100 | 1,120 | 2,500 | Arithmetic Growth Rate Ten Years |  |  |  |  |  |  |
| 2 | 1,040 | 1,080 | 1,120 | 1,160 | 1,200 | 1,240 |  |  |  |  |  |  |  |  |
| 3 | 1,060 | 1,120 | 1,180 | 1,240 | 1,300 | 1,360 | 2,000 |  |  |  |  |  |  |  |
| 4 | 1,080 | 1,160 | 1,240 | 1,320 | 1,400 | 1,480 |  |  |  |  |  |  |  |  |
| 5 | 1,100 | 1,200 | 1,300 | 1,400 | 1,500 | 1,600 | 1,500 |  |  |  |  |  |  |  |
| 6 | 1,120 | 1,240 | 1,360 | 1,480 | 1,600 | 1,720 |  |  |  |  |  |  |  |  |
| 7 | 1,140 | 1,280 | 1,420 | 1,560 | 1,700 | 1,840 | 1,000 |  |  |  |  |  |  |  |
| 8 | 1,160 | 1,320 | 1,480 | 1,640 | 1,800 | 1,960 |  | 2 | 3 |  | 56 | 78 | 9 | 0 |
| 9 | 1,180 | 1,360 | 1,540 | 1,720 | 1,900 | 2,080 |  |  |  |  |  |  |  |  |
| 10 | 1,200 | 1,400 | 1,600 | 1,800 | 2,000 | 2,200 |  |  |  | 6\% | -8\% | -10\% | - 12\% |  |

Table 14

Compound Annual Growth Rate (CAGR)

|  | 1,000 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Yr | $2 \%$ | $4 \%$ | $6 \%$ | $8 \%$ | $10 \%$ | $12 \%$ |
| 1 | 1,020 | 1,040 | 1,060 | 1,080 | 1,100 | 1,120 |
| 2 | 1,040 | 1,082 | 1,124 | 1,166 | 1,210 | 1,254 |
| 3 | 1,061 | 1,125 | 1,191 | 1,260 | 1,331 | 1,405 |
| 4 | 1,082 | 1,170 | 1,262 | 1,360 | 1,464 | 1,574 |
| 5 | 1,104 | 1,217 | 1,338 | 1,469 | 1,611 | 1,762 |
| 6 | 1,126 | 1,265 | 1,419 | 1,587 | 1,772 | 1,974 |
| 7 | 1,149 | 1,316 | 1,504 | 1,714 | 1,949 | 2,211 |
| 8 | 1,172 | 1,369 | 1,594 | 1,851 | 2,144 | 2,476 |
| 9 | 1,195 | 1,423 | 1,689 | 1,999 | 2,358 | 2,773 |
| 10 | 1,219 | 1,480 | 1,791 | 2,159 | 2,594 | 3,106 |

Compound Annual Growth Rate (CAGR)


Two percent returns may seem unrealistic, but the compound annual inflation rate over the last hundred years is $6.66 \%$. If returns are at $8.66 \%$, we have $2 \%$ real returns. It is even worse if inflation is at $8 \%$ and returns are at $6 \%$. Consistent higher returns are worth striving for. Higher yields make an increasingly significant difference for each additional year.

The dividends can be added to the value of the investment as shown in Table 8, creating yields directly comparable to the CAGR shown in Table 1 for price appreciation. Yield therefore looks the same as return. In our example, instead of the $\$ 2,000$ after ten years for the Yield on Cost, we have the $\$ 2,594$ or a significant difference. When you see yield in a report, how was it calculated? Both examples are a yield of $10 \%$.

A yield of $10 \%$ when reinvested is going to have the same return from income alone as a compound annual growth rate (CAGR) of $10 \%$ from price related gains. When a position is called, such as at $\$ 25$ for preferred stocks, the return or gain on price is added to the income return. The difference between a CAGR for annual price-appreciation and for yield is that the price-appreciation calculation is hypothetical. The position probably was not purchased at the beginning of the period and not sold at the end.

Reinvesting in the same position increases the shares and value for that position and thus the denominator in a return calculation, just like for price appreciation the denominator grows for each period. In both cases, after several time periods, a compound annual growth rate (CAGR) of 10\% will be the same for dividend yields as for returns based on market pricing of stocks intended for price appreciation.

Not only is it unrealistic to expect an investment for price gains to have a consistent compound annual growth rate, to evaluate fixed income based on false assumptions is unrealistic. Calculating yield based on dividend reinvestments is theoretically accurate; it is not feasible in practice.

## Disadvantages of reinvesting in the same position include:

a. The reinvestments will likely not be at the original cost, as prices vary for the same position. For projecting future returns, assuming at what price a dividend could be reinvested is making judgments about unknown future prices and availability and is not a very reliable and accurate way to evaluate a position or future dividends. Price changes are a hard contingency to avoid. The best resolution is to be aware of assumptions, such as future purchases being made at original cost.

The positive part of price variation is that it is a form of dollar cost averaging. The regular dividend reinvestments smooth out market volatility as positions are acquired at different points in the cycles of market oscillations.
b. The same investment may not be the best investment.
c. The reinvestment either needs to purchase fractional shares, or there is slippage in some of the dividend not being reinvested.
d. An additional administrative challenge is that the additional shares create a new basis for that part of the position, creating complexity in measuring how many dollars were invested for how long. If the calculation uses the original cost, ignoring the cost or basis of additional purchases, this preempts any cost per share analysis, as each reinvestment has its own basis and length of holding. In effect, reinvested share purchases have a cost, but in the return calculations reinvestments have a zero cost since they were automatically acquired in the form of dividends. If the number of shares goes up and then the market goes down, the overall returns will be distorted.
e. Any buying or selling that changes the number of shares in a position, such as for withdrawals or for more attractive investments, alters the basis and makes accurate yield computations more complex.

## 4. Reinvesting in different positions

Since significant disadvantages surface when reinvesting in the same position, we will explore the implications of reinvesting in attractive and timely new positions.
Typically, dividends are aggregated from multiple positions, and when cash is sufficient to buy another position of similar size, a new investment is made. This keeps the dividend money invested and in optimum positions, but within another investment. While this seems straightforward, a diagram contrasting the distinction might be helpful for the discussion of how to account for dividend returns.

In the A part of the diagram all positions are reinvested in the original investment.

In the B diagram dividends are reinvested in new positions, each reinvestment shown by a line. Only arrowed lines for the first generation of dividends are shown. Obviously, each new investment spawns additional investments in a geometric pattern, and are consolidated with dividends from other positions. If the dividends are reinvested in different preferred stocks, the result will be reflected not in the original position, but in the combination of the original position and all descendants, which could be several generations as investments purchased from dividends also yield dividends which are reinvested in additional stocks.
Administratively, reinvesting in new positions is the practical way to manage reinvesting fixed income. Since the returns then flow from a new investment, what are various useful and practical ways to look at returns?

If we want to measure the contribution of the original position in the eighth year it has been held, the combined yield for that stock and all its descendants will be much higher than an arithmetic yield. Since we know that each position yields CAGR returns, it is relatively easy to set up a power function in Excel to calculate accrual returns for each position based on original cost, original yield
and the duration of the holding. The spreadsheet calculates the days held, which is then converted to a fractional year for use in the power formula to derive the yield for a geometric increase. Note that his is a hypothetical return for the position assuming that dividends had been reinvested in the same position. It may be relevant for analyzing portfolio performance for different time periods and different markets, or for comparing on other dimensions. It does not reflect the actual cash flow.

An accrual look at the next year is more accurate than calculations based on history since dividends are only paid quarterly and may be paid for positions no longer held, or may not yet have been paid for positions purchased recently. Using the CAGR calculation enables an accrual number identical to the accrual of investing dividends in the original position at the original cost and is directly comparable to a CAGR from price appreciation.

Looking at the bottom arrowed line in the B part of Table 15, the cash flow reality is that stripping off the dividends as they are paid has the same effect for each position as ignoring dividends, converting geometric returns to arithmetic returns for that position. This portion of the returns are the same as for Yield on Cost or Out of Pocket calculations. If the returns are withdrawn and reinvested in new positions, they do not add to the value of the position. A Yield on Cost calculation ignores any reinvestment of dividends, which is accurate for the position but not the portfolio. When reinvesting in new positions, a geometric increase in returns is reflected in the size of the portfolio and its returns, leaving the individual position to reflect arithmetic returns with yields being the same each year.

In summary, at the level of the individual position, the CAGR calculation reflects progressive yield increases which reflect on the position but have limited utility since the dividends were reinvested in other positions. The progressive yields are measured at the portfolio level rather than the level of the specific position. The Yield on Cost based on stripping off the dividends gives a more accurate result in terms of dividends from that position but fails to account for the return on reinvested dividends. In practical terms, an arithmetic mean and a geometric mean are the same for the first period, so if all dividends from the original position and all dividends from reinvested dividends are used to buy new positions, the actual yields at the level of the individual position will be arithmetic and it doesn't matter. To get an accurate picture of CAGR one must look at the portfolio level.

## Portfolio Returns

How do we calculate portfolio returns? If we divide the portfolio dividends by the original cost for the portfolio, the returns will be arithmetic since we have arithmetic returns in each position. The value of the portfolio has grown geometrically with the additional positions, but the rate of return for each position has not grown geometrically and remains tied to the variations in market pricing available at purchase.

When looking at portfolio returns, averages of individual positions are going to be different from and less accurate than dividing the total portfolio (accrued or actual) dividends by the total costs. For an average of positions to have meaning one would need to adjust for varying position sizes. Even if giving an arithmetic mean, taking the Yield on Cost from the portfolio totals for dividends and costs is going to be close to accurate as the new positions tend to keep the average age of positions fairly low. Over ten years, $48 \%$ will have been held two years or less. Evaluating quarterly changes in total costs or total dividends is going to be impacted by money being added or withdrawn.

Another approach is to apply a power function and CAGR projection to a total portfolio just as done for individual positions. To see how that works, I used the Wenzel Analytics experience. Plugging in the $9.4 \%$ Yield on Cost, the 1.52 average years' duration (maximum 8.8, median 1.57) and the total
costs, the CAGR is $\mathbf{1 0 . 8 3 \%}$ for the past year. For the next year the CAGR is $\mathbf{1 1 . 8 5 \%}$. The results would be more accurate if the 1.52 average years' duration were adjusted for the varying holding periods and position sizes, but probably not significantly so. The average duration is minimized by new clients, shifting allocations to preferred stocks, and reinvesting dividends in new positions.

In practice there is slippage from when dividends accumulate and are combined with other dividends to be reinvested.

Since dividends sometimes are received from stocks sold prior to the reporting period, and newly acquired positions have a cost figure but may not yet have dividend receipts, it is more accurate (and useful) to calculate future yield than past cash flow. Results based on historical dividends received if calculated on an annual basis would not reflect when during the year the dividends were received. Using the accrual method used for the results presented here attributes the dividends equally to each day. An accrual return will be slightly different than a return based on cash flow.

## Data Illustration: Contrasting Methodologies for Yields and Income Returns

Differing methodologies and their implications were described above. What do the respective calculations look like when given numbers?

When comparing Table 9 which ignores reinvested dividends to the price appreciation investment shown in Table 1, we see the $10 \%$ yield producing $\$ 2,000$ while the $10 \%$ compound annual growth rate (CAGR) resulted in $\$ 2,594$, or $30 \%$ more. The difference of ignoring the reinvested dividends is shown in Table 16. The growth return rate each year shown in Table 1 is based on the prior cumulative price appreciation. The yield on cost is calculated from the original investment. The longer a fixed income position is held, the more the yield on cost will trail a return having the same number. A yield calculation that includes reinvested dividends is a CAGR return and directly comparable to the CAGR for price appreciation. However, it has other challenges discussed above.

Table 16 shows the contribution of reinvested dividends in a slightly different format than shown in Table 8. Showing just the dividend progression in Table 16 makes the contrast between Yield on Cost and Yield based on Reinvested Dividends more vivid.

Table 16


Table 16 can also be viewed as a comparison between calculating yield as Yield on Cost (ignoring yield on reinvested dividends) and capital gains if sold at year ten with a CAGR 10\% return. The $10 \%$ calculated as Yield on Cost doesn't compare to the capital gains at a $10 \%$ return.

## Comparing Gains to Income

## Comparing Returns for Income and Gains

In Table 17 we have added capital gains to the chart in Table 8.
Table 17.
Income and Gains-Example


The growth each year for price appreciation is based on the preceding changes in valuation, while the income Yield on Cost is in orange on Table 16 and Table 17. The dramatic difference is that for the valuation-based investment the investor has nothing to show for it until the sale, while the income investor has annual (or quarterly) income. If a non-income stock had a $10 \%$ CAGR, picture it without the middle two gray and orange stacked bars in Table 17.

## Comparing Reliability

There is a significant difference in the reliability of results based on price and results based on fixed income. Table 1 assumes a consistent CAGR return over the ten years. What if we looked at some actual S\&P 500 returns using the annual returns from Table 2?

The table below gives returns from three successive ten-year periods. The \%Chg is for the S\&P 500 and the chart in Table 2. The Dollars columns converts that percent change into dollars for a $\$ 1,000$ investment. The CAGR takes the ten-year result of the successive annual returns and smooths it to a single annual rate for the ten years. The Var column gives the dollar variance or difference between the dollars based on the annual rates and the CAGR rate.

Table 18
Ten-Year Gains

|  | Cost \$1,000 |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2013-2022 |  |  |  | 2003-2012 |  |  |  | 1993-2002 |  |  |  |
|  | A | B | C | D | E | F | G | H | 1 | $J$ | K | L |
| Year | \% Chg | Dollars | CAGR | Var* | \% Chg | Dollars | CAGR | Var* | \% Chg | Dollars | CAGR | Var* |
|  |  |  | 10.56\% |  |  |  | 7.03\% |  |  |  | 9.25\% |  |
| 1 | 13.52\% | 1,135 | 1,106 | 30 | 28.36\% | 1,284 | 1,070 | 213 | 9.97\% | 1,100 | 1,093 | 7 |
| 2 | 1.38\% | 1,151 | 1,222 | -72 | 10.74\% | 1,421 | 1,146 | 276 | 1.33\% | 1,114 | 1,194 | -79 |
| 3 | 11.74\% | 1,286 | 1,352 | -66 | 4.83\% | 1,490 | 1,226 | 264 | 37.20\% | 1,529 | 1,304 | 225 |
| 4 | 11.77\% | 1,437 | 1,494 | -57 | 15.61\% | 1,723 | 1,312 | 410 | 23.82\% | 1,893 | 1,425 | 468 |
| 5 | 21.61\% | 1,748 | 1,652 | 96 | 5.48\% | 1,817 | 1,405 | 413 | 31.86\% | 2,496 | 1,557 | 939 |
| 6 | -4.23\% | 1,674 | 1,827 | -153 | -36.55\% | 1,153 | 1,503 | -350 | 28.34\% | 3,203 | 1,701 | 1,502 |
| 7 | 31.22\% | 2,197 | 2,020 | 177 | 25.94\% | 1,452 | 1,609 | -157 | 20.89\% | 3,872 | 1,858 | 2,014 |
| 8 | 18.01\% | 2,592 | 2,233 | 359 | 14.82\% | 1,667 | 1,722 | -55 | -9.03\% | 3,522 | 2,030 | 1,492 |
| 9 | 28.47\% | 3,330 | 2,469 | 861 | 2.10\% | 1,702 | 1,843 | -141 | -11.85\% | 3,105 | 2,218 | 887 |
| 10 | -18.01\% | 2,730 | 2,730 | 0 | 15.89\% | 1,973 | 1,973 | 0 | -21.97\% | 2,423 | 2,423 | 0 |

[^0]The conclusion I would draw is that there is considerable variation year to year, but if one is willing to just let things be for ten years, the annual returns for a sample of three are between $7.0 \%$ and $10.5 \%$.

In Table 19 we extend that analysis to the years since 1928 shown in Table 2. The annual returns of the S\&P 500 are a line rather than in bars. The volatility and uncertainty are obvious. The orange line reflects a CAGR moving ten-year holding period. It smooths out the returns considerably. However, even after ten years, it went negative or slightly below three times which is a long time to not have gains, ignoring inflation (CAGR 6.6\% last 100 years). The range in CAGR returns over ten years between $-4.6 \%$ and $20.7 \%$ is still wide. For half of these years, returns were lower the next ten years than they were the preceding ten years. We can't judge the future ten years by the past ten years. The trend appearance is a result of each annual return being reflected ten times in the ten-year rolling CAGR. Think of it as a ten-year exponential moving average.
Table 19


As you can see, I have tried to find order or patterns in the market returns reflected by Tables 2 and 19, with only limited success shown in Table 3. In presenting Tables 2 and 19 to a small group of investors, I found that they similarly kept trying to find predictability between time periods. I find it amazing how we persist in looking for patterns when none are to be found, even to the extent of deriving patterns from data sets too small for statistical significance. Similarly, I find experienced and professional investors using and promoting criteria and screens which appear intuitively logical but lack empirical verification. Watch most any chapter presentation on the AAll website.

Both income and stocks purchased only for price appreciation are vulnerable to market volatility, but in different ways and to different degrees. The S\&P 500 returns in Table 19 are for income plus capital gains or losses. Income and price appreciation are not separated. Overall, the dividend contribution is minimal. For 2021, the dividend yield was $1.3 \%$ compared to a $4.3 \%$ dividend yield for the long run average of the index (Investopedia).

Preferred stocks with much higher dividend yields than the S\&P 500 are not large enough to be included in the S\&P 500 index. Consequently, the comparison here between price appreciation as reflected by the $S \& P 500$ is a different sample than preferred stocks, rather than preferred stocks being a subset. Preferred stocks as an example of fixed income have a capital gains component and an income component, which can be separated and compared.

How do capital gains work for preferred stocks? The big difference between preferred and common stocks is that for preferred stocks, we know the selling price will be par ( $\$ 25$ ) unless we choose to sell when prices are above that level, or for other reasons choose to not wait for the call. If the stock is purchased at par rather than at a discount, we know there will be no capital gains when the position is called. If bought at a discount, for example $\$ 20$, we know there will be a $\$ 5(25 \%)$ gain upon call. If the
call is in one year, the capital gains are $25 \%$. If the call is in ten years, the arithmetic annual gains are $2.5 \%$. This known selling price limits the price volatility, especially if callable or the call date is imminent.

How do long-term numbers from capital gains compare to those from income once we have comparable metrics to measure return? Tables 2,18 and 19 give a picture for price appreciation stocks based on history, as do the simulation Tables 6 and 7. Table 17 gives a picture for income, assuming a $10 \%$ yield and $10 \%$ gain, which is consistent with Wenzel Analytics returns identified above. The general historical record on preferred stocks has returns about half the Wenzel Analytics experience. The difference is primarily accounted for by institutional requirements for trading volume and using industrial grade as a measure of presumed quality. Preferred stocks confuse both the buyer of common stocks and of bonds, making it somewhat of a neglected and little understood asset class.

In general income-alone from preferred stocks can be expected to be comparable to very long-term (more than ten years) price appreciation from common stocks. Capital gains from preferred stocks can be considered an additional bonus. The total capital gains from preferred stocks can be relatively certain since they are called at par, while the timing of a call and annual CAGR from gains are uncertain.

Beyond the numbers, comparisons are relevant for certainty of returns, relatively risk-free access to accumulating dividends, and what is required to manage the respective investments.

The portfolio volatility for income stocks is much less than for price appreciation strategies because the volatility only relates to the reinvested dividends and not the whole portfolio.

One big difference between investing for price appreciation and investing for income is that the income can preempt the need to withdraw funds by selling at low prices. The fixed-income protection against needing to sell in order to access funds affects not having to sell investments for price appreciation as well as income investments.

## Investments with Both Dividends and Gains

This write-up has been about pure types, fixed income and stocks for price appreciation. The reality is that most but not all investments are a hybrid to one degree or another.

Unless an emergency makes one desperate for cash, preferred stocks should never have a loss except for an unforeseen bankruptcy. (With a thousand positions, I've seen one. Some are threatening but now coming back. I've been buying a few preferreds for under $\$ 2$ that have strong insider buying and where the company is getting big contracts from big players in their industry - probably pointing to an acquisition. At an eventual $\$ 25$ call, that is a good gain even if it doesn't happen this year.)

How do you separate income return from price appreciation for a stock with a $3 \%$ dividend? If it is a short-term return, one can subtract the dividend to the profit or loss, assuming the holding period captured the dividend. The market usually adjusts a stock price relative to an impending or recent dividend, although the impact tends to be limited to the immediate days prior to and following a dividend. Some brokers calculate accrued dividends if you want to get very precise. If it is a long-term holding, to get an accurate return one would need to do a yield calculation as described above. I suspect that is rarely done and that consequently investors with long-term holdings of companies paying dividends of less than $5 \%$ do not adequately account for the balance between income and gains on a specific investment. One may look at the dividend rate, but declared rates change and the reported dividend yield is adjusted to price.

## The Psychology

Most of us for most investments think in terms of either income or gains/losses. For rental property I calculate net cash flow and from that an annual cap rate and cap rate based on original cost. Until the property is sold appreciation or depreciation is an interesting speculation, but just that. I consider it a
bonus. I similarly evaluate my money management business looking at net profit. I ignore valuation and of course changes in valuation since I'm not interested in ever selling the practice. I don't know or care if it is even possible. If one is holding a bond or preferred stock for long-term income, it is the income that matters. Price variations between acquisition and sale are irrelevant. Tracking market price changes only introduces irrelevant anxiety. With preferred stocks purchased for less than par, we know that it either will be called for a gain at $\$ 25 /$ share, or we will choose to sell it for a gain, especially if prices move above $\$ 25 /$ share. It is hard to wean ourselves from reporting and looking at price data, since it is part of nearly all reporting. Income investors familiar with buying bonds having a fixed maturity are familiar with thinking this way, especially if the bonds are relatively illiquid and price quotes are less readily available.

Investors tuned to gains and losses are focused on price variation. Income investing is a different paradigm and different way of thinking and conceptualizing returns. The familiar context is for dividends to be thought of as a vague bonus, relatively inconsequential to decisions about the merit of buying or holding an investment. Even if bought for income, they still watch and worry about price oscillations.

The fact that preferred stocks are called stocks when they are more like debt confuses both the bond and stock investors. Consequently, preferred stocks become somewhat of a neglected asset class, not properly understood for what they are, how they work and how to work them.

Fixed income preferred stocks tend to be boring relative to stocks owned for price appreciation. For the manager of income investments, the buying decision is much more administrative since one mostly needs to calculate and compare known future results. Stocks are ranked by Current Yield and then evaluated by Yield to Call and factors such as being cumulative, domestic, already owned or own another position issued by the same company, fixed/float features, and the quality of the underlying or issuing company. The latter is mostly revealed by checking the price charts of the parent company. The common stock doesn't have to be going up; it just has to not be dropping precipitously below market indexes. I think of it as being a cook working from a recipe.

As for selling preferred stocks, it is mostly a matter of waiting for a call. If prices rise above par, it might be time to take gains if the Yield to Call doesn't compensate for the potential loss of a call at par rather than the current higher price. I scan a list having conditional formatting that alerts me to a position trading above par and having a Yield to Call below 5\%. I scan the underlying common stock charts for prices falling significantly beyond market declines to alert me to a possible bankruptcy. The scan is easily automated. As cash from dividends accumulates in the respective account, I go through a current buy list and buy positions not already held in that account. It is all very routine and administrative.

The ongoing review and management decisions for income positions such as preferred stocks are mostly a matter of buying decisions rather than selling decisions, and then the decisions are much easier because of the specific criteria to apply. The strategy and systems put in place enable a feeling of comfort and control.

Preferred stock management is very much in contrast to buying stocks for price appreciation. For the manager, price appreciation investments require decisions as to what to buy, when to buy and when to sell. The selling decision is particularly crucial for taking returns. These are difficult decisions, given the inherent uncertainties. It is hard to have rules for these decisions that work predictably. I think of such investors as chefs, making creative and one-off decisions.

I easily become enmeshed in the stock's story and my reason for buying it. I watch the charts, wondering if momentum up or down will continue or if there will be a reversal. I weigh daily, weekly and monthly candlesticks to zoom my time frame in and out. I look at how the stock is graded by analysts and various investor services. How does it fit in sector rotations? Is the market moving toward small cap or away from large cap tech? Should I sell a position and buy the stock of an upcoming
competitor? There is no end to the attention and anxiety that can be invested in any one position. While sometimes I like excitement, looking at charts and having a story to tell myself, a partner or friends, I don't want too much of it - especially with other people's money.

The extent to which we put our energies into buying fixed income or buying stocks for possible gains can be very much dependent upon what we have in life that occupies our attention and what we need. Too often our decisions are emotionally driven; what we think is rational is more of a rationalization.

Some of us are more left-brain and analytical. We like to cooly figure things out. (See any of that here?) Others are more right-brain, looking for excitement and energizing changes. Often it is good advice to use the left-brain for the core or major part of our investments while carving out an allowance of play money for the more right-brain speculative bets. Frequently the investor is a bigger variable in returns than are the investments.

Fixed income investments with regular dividends or interest payments are more convenient for investors needing income to meet lifestyle expenses. Income automatically collects in the account and can be automatically withdrawn each month. For the non-income investor, cash flow for withdrawals requires continual decisions about what to sell.

## Conclusions

This analysis was precipitated by wanting a return calculation for yields that could be used to match returns from gains. I was and am primarily concerned with deciding how to divide allocations between fixed-income and price-dependent investments, which are inherently speculative. What appeared to be missing was accounting for the discounted time value of future dividends.

The big difference is that dividends are cash to be reinvested. A stock going up in value the same amount does not create money to be reinvested.

I've learned or confirmed several things.

1. The primary distinction of income returns is the certainty of annual returns. As for the amount of return, in general income-alone from preferred stocks can be expected to be comparable to very long-term price appreciation from common stocks (more than ten years).
2. Capital gains from preferred stocks can be considered an additional bonus if purchased at a discount. The total capital gains from preferred stocks can be relatively certain since they are called at par, while the timing of a call and annual CAGR from gains are uncertain.
3. Price appreciation returns vary significantly and are not predictable. Calculating probability is probably more important than calculating returns, at least for returns based on a limited number of years.
4. Income is much more predictable than investments related in any way to price. While all income investments are dependent upon a purchase price and an eventual sale or call price, there is value in metrics that separate the intervening dividend returns from total returns that include price volatility.
5. Price changes impact the whole position for price appreciation strategies. For income strategies price changes only affect the purchase of dividend reinvestments. Since the discount from par varies, the reinvesting in new positions has a dollar averaging effect, smoothing the impact of market volatility on both eventual price appreciation and on yields.
6. Income can preempt the need to withdraw funds by selling at low prices.
7. Fixed income, by definition, is a compound annual growth rate investment (CAGR). That fact provides an important benchmark for comparing income to price appreciation. It also provides a convenient way to calculate future returns based on costs, a known yield and formulas for calculating geometrically increasing returns by the duration of the holding.
8. Reinvesting dividends into the same position has several practical impediments. Using the mathematical power function enables calculating CAGR reinvested dividend returns for a position even if a dividend is reinvested along with dividends from other positions in new and better investments.
9. Because dividends are continually stripped off rather than added to the original investment, dividend cash flow from each investment maintains a straight-line arithmetic growth pattern. The geometric CAGR for the portfolio is reflected in new positions adding to the size of the portfolio.
10. The most satisfactory way to calculate portfolio returns is to apply a power function and CAGR history or projection to a total portfolio in the same manner as done for individual positions.

Testing on available data, doing a CAGR power calculation for the portfolio just as done for individual positions revealed a 1.4\% higher return than the Yield on Cost for the preceding year, and $2.4 \%$ higher yield for the next year.

It is not feasible to calculate aggregate portfolio returns from individual position returns. Any computations based on position data get complex and less accurate. Numbers need to be adjusted for varying position sizes. Averages don't always reflect the distribution. CAGR for individual positions reflect value that is no longer tied to the position but has been reinvested.

Any quarterly or annual comparisons of total cost or total dividends would need to account for withdrawals or money added to the portfolio.
11. Some preferred stocks have qualified dividends with the same income tax rate as capital gains. Preferred stocks issued by REITs and many other companies do not have qualified dividends, making them a relative advantage for the lower income investor.
12. It is more administrative and almost boring to manage income investments, in contrast to the tension of speculating with when to buy, hold and sell price-appreciation investments.
13. The initial concern about the periodic and thus delayed dividend income (discounted future value), when calculated, turned out to be an obvious and primary advantage of fixed income. First off, doing the actual calculations revealed that the impact on yield is modest. The second, and most important realization, is that if we balance things out by applying a discounted future value to price-appreciation dependent investments, the discounted future value, depending on the rates selected, can wipe out all the gains. We calculate a discounted future value on fixed income because we can. We then lower the value of the fixed income. We don't calculate a discounted future value on price-based investments because it is only a guessing game. I decided to leave sleeping dogs lie and ignore the discounted future value calculations.

Getting a return calculation for fixed income that gives an appropriate comparison to unrealized gains or losses is more complicated than one would think. The deeper the analysis and the more precision required, the more complex it becomes. Just because it is a number does not mean that two numbers mean the same thing. Getting an approximate comparable return number is important in making an allocation decision between owning debt and owning equity. Other more personal variables also come into the allocation decision.

What is it worth to have the certainty of cash accumulating along the way? Is that an advantage or a nuisance requiring work to reinvest? What is it worth to not be dependent upon market fluctuations for when one needs or chooses to make withdrawals? Is the income sufficient to cover withdrawal needs? Would the relatively certain income return exceed the possibility of investing in an explosive stock?

A very big difference is that with price-appreciation we are reporting on the past and making assumptions about the future. For fixed income there is no need to look at the past since it has passed, we can't do anything about it, it may not have much if anything to tell us about the future and we can plan with near contractual certainty what the income will be for the next year. Is a bird in the hand worth two in the bush? If not, what is it worth?

Having price-appreciation numbers for returns to match against returns from income helps in deciding between price-appreciation investments and income investments.

## Appendix: Context and Background for Understanding Income

A. Investments are of two basic types. (Like ham and eggs, a one-off or continuous).

1. Those intended for gain resulting from a price increase between purchase and sale (usually equities)
2. Those producing income (usually from debt)
B. Investor allocation should rationally balance these two pure types.
3. Even for investments over a given period having components of each type, it is useful to look at the price appreciation component separately from the income component.
4. Common metrics are needed to compare the results of these two investment types.
C. Price-dependent investments are measured by return.
5. Return is a percent change in price or valuation between the beginning and end of a period.
6. Return is usually annualized (made equivalent to an annual return), except for returns covering less than one year.
7. Returns are volatile because prices change.
D. Income investments calculate yield in different ways.
8. Each year Yield on Cost (dividends over the period divided by cost) stays the same value since earlier dividends are ignored. Ignoring the timing of dividends and any consequent yield or returns from the dividends results in an arithmetic, lower number for yield which should not be compared to price-dependent returns. A 10\% yield based on a straight-line arithmetic increase is not the same as a $10 \%$ return based on progressive increases.
9. Yield Including Reinvested Dividends
a. Gives a number comparable to price-based returns.
b. Projections assume the dividends will be reinvested at the same price and dividend rate as the original investment. Rarely is this possible, making for discrepancy to actual returns.
c. In practice, dividends from multiple investments are typically combined for the purchase of the most attractive investment, making accrual results of the original dividend hypothetical rather than based on cash flow. Actual results are reflected at the portfolio level by reinvested dividends, reiterated through multiple generations of reinvestments.
E. Overview contrasting the mathematics for converting yield to return.
10. A price-dependent return is calculated by adding to the numerator the difference between the end and beginning valuations and dividing by the beginning valuation.
11. A yield including reinvested dividends has a similar calculation, adding dividends and the value of reinvested dividends to the numerator. The result matches the compound annual growth rate (CAGR) used for price-dependent returns to get a single return number covering multiple years or periods with varying returns.
12. The yield on cost calculation doesn't change the numerator or denominator in successive years (periods).

[^0]:    * Var: Variance or difference betw een annual rates in dollar effects and the geometric CAGR over the ten years.

